



Math in Focus: Singapore Math National Institute
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Welcome!



Houghton
Mifflin
Harcourt

Kelly Snyder, M. Ed.

National *Math in Focus* Specialist

An In-Depth Look by Grade Level

Grades 2 and 3

Big Ideas in Grades 2 and 3:

Mathematical Proficiency

Pacing in terms of importance

Understanding through assessing

“I am slow to learn and slow to forget that which I have learned.

My mind is like a piece of steel; very hard to scratch anything on it and almost impossible after you get it there to rub it out.”

Abraham Lincoln to his friend Joshua Speed,
quoted in a letter to *The New Yorker*

Learn portion of the lesson is not to be taken lightly...

10-15 minutes of discussion that is defined by extended engagement with a small number of tasks.

Yeap Ban Har, 2012

Working on the Math

Skills Trace and Challenging Problems

Grades 2 and 3

Frame for our work: Theory and Practice

Let's Discuss:

- Important Content and How it Develops
- Solving problems: The Challenges
- Solving problems: How to Scaffold

Number and Operations

Content that should dig deep



Place Value and Addition/Subtraction

Common Core State Standards

What does CCSS have to say?

Operations and Algebraic Think

Represent and solve problems

1. Use addition and subtraction situations of adding to, taking from, and comparing, with unknowns in all positions, drawings, and equations with a symbol for the unknown to represent the problem.²
2. Solve word problems that call for addition of whole numbers whose sum is less than or equal to 100, and equations with a symbol for the unknown.

Work with addition and subtraction

7. Understand the meaning of the equal sign in equations involving addition and subtraction. *of the following equations are true: $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$.*
8. Determine the unknown whole number in an equation relating three whole numbers. *unknown number that makes the equation true: $? + 11 = 20$, $5 = \square - 3$, $6 + 6 = \square$.*

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
 - a. 100 can be thought of as a bundle of ten tens — called a “hundred.”
 - b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by 5s, 10s, and 100s.
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
9. Explain why addition and subtraction strategies work, using place value and the properties of operations.³

GR 2

Operations and Algebraic Thinking

3.OA

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.³
9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Number and Operations in Base Ten

3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.⁴

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.



Numbers to 1,000

Read, Write,

Place Value

Compare

Order and

Subtraction up to 1,000

Subtraction without regrouping

Subtract the ones.
Subtract the tens.
Subtract the hundreds.

$$\begin{array}{r} 876 \\ - 234 \\ \hline 642 \end{array}$$

Check using addition.
If $876 - 234 = 642$,
then $642 + 234$
should equal 876.

$$\begin{array}{r} 642 \\ + 234 \\ \hline 876 \end{array}$$

The answer is correct.

Subtraction with regrouping

Regrouping in tens and ones.

Regroup.

$987 = 9$ hundreds 8 tens
 7 ones
 $= 9$ hundreds 7 tens
 17 ones

$$\begin{array}{r} 9\overset{2}{8}7 \\ - 129 \\ \hline 858 \end{array}$$

Check using addition.
If $987 - 129 = 858$,
then $858 + 129$ should
equal 987.

$$\begin{array}{r} 8\overset{1}{5}8 \\ + 129 \\ \hline 987 \end{array}$$

The answer is correct.

Regrouping in hundreds
and tens.

$$\begin{array}{r} 9\overset{1}{4}6 \\ - 253 \\ \hline 693 \end{array}$$

Regroup.

$946 = 9$ hundreds 4 tens 6 ones
 $= 8$ hundreds 14 tens 6 ones

Check using addition.
If $946 - 253 = 693$,
then $693 + 253$ should
equal 946.

$$\begin{array}{r} 693 \\ + 253 \\ \hline 946 \end{array}$$

The answer is correct.

Regrouping in hundreds,
tens, and ones.

$$\begin{array}{r} 6\overset{1}{0}\overset{1}{0} \\ - 487 \\ \hline 113 \end{array}$$

Regroup.

$600 = 6$ hundreds
 $= 5$ hundreds 10 tens
 $= 5$ hundreds 9 tens
 10 ones

Check using addition.
If $600 - 487 = 113$,
then $113 + 487$ should
equal 600.

$$\begin{array}{r} 113 \\ + 487 \\ \hline 600 \end{array}$$

The answer is correct.

Solve real-world subtraction problems.

A bakery sells 347 loaves of bread on Sunday.
It sells 168 fewer loaves of bread on Monday.
How many loaves of bread does the bakery sell on Monday?

$$347 - 168 = 179$$

$$\begin{array}{r} 3\overset{1}{4}7 \\ - 168 \\ \hline 179 \end{array}$$

The bakery sells 179 loaves of bread on Monday.

She sells 421 cookies.

$$\begin{array}{r} 276 \\ + 145 \\ \hline 421 \end{array}$$

ON YOUR OWN

Go to Workbook A:
Chapter Review/Test,
pages 69–72

ON YOUR OWN

Go to Workbook A:
Chapter Review/Test,

Numbers to 10,000

Read, Write and Count

9,745

- Expanded form:
 $9,000 + 700 + 40 + 5$
- Word form:
nine thousand, seven hundred forty-five
- Standard form: 9,745

Count by

- ones: 3,928 3,929 3,930
- tens: 2,096 2,106 2,116
- hundreds: 813 913 1,013
- thousands: 4,126 5,126

Place Value

	9	7	
9 thousands	←		
7 hundreds		←	
4 tens			←
5 ones			←

Addition up to 10,000

Without Regrouping

$$2,315 + 1,231 = 3,546$$

$$\begin{array}{r} 2,315 \\ + 1,231 \\ \hline 3,546 \end{array}$$

- Step 1** Add the ones
5 ones + 1 one
= 6 ones
- Step 2** Add the tens
1 ten + 3 tens
= 4 tens
- Step 3** Add the hundreds
3 hundreds
+ 2 hundreds
= 5 hundreds
- Step 4** Add the thousands
2 thousands
+ 1 thousand
= 3 thousands

Subtraction up to 10,000

Without Regrouping

Subtract the ones. $4,663$
Subtract the tens. $- 1,231$
Subtract the hundreds. $3,432$
Subtract the thousands.

Check using addition.
If $4,663 - 1,231 = 3,432$,
then $3,432 + 1,231$ should equal 4,663.

$$\begin{array}{r} 3,432 \\ + 1,231 \\ \hline 4,663 \end{array}$$

With Regrouping

Regroup.

$$\begin{array}{r} 9,876 \\ - 7,877 \\ \hline 1,999 \end{array}$$

$= 9$ thousands 8 hundreds
7 tens 6 ones
 $= 9$ thousands 8 hundreds 6 tens
16 ones
 $= 9$ thousands 7 hundreds 16 tens 16 ones
 $= 8$ thousands 17 hundreds 16 tens
16 ones

Check using addition.
If $9,876 - 7,877 = 1,999$,
then $1,999 + 7,877$
should equal 9,876.

$$\begin{array}{r} 1,999 \\ + 7,877 \\ \hline 9,876 \end{array}$$

Regroup.

$$\begin{array}{r} 5,000 \\ - 4,321 \\ \hline 679 \end{array}$$

$= 5$ thousands
 $= 4$ thousands 10 hundreds
 $= 4$ thousands 9 hundreds 10 tens
 $= 4$ thousands 9 hundreds 9 tens
10 ones

Check using addition.
If $5,000 - 4,321 = 679$,
then $679 + 4,321$
should equal 5,000.

$$\begin{array}{r} 679 \\ + 4,321 \\ \hline 5,000 \end{array}$$

Skip Counting:

- Develops fluency of a number word sequence
- Must be fluent both forward and backward
- Makes mental strategies more accessible
- Does not account for quantity

Skip Counting Scenarios

- Forward/backward by ones
- Forward/backward by tens
- Forward/backward by _____
- Forward/backward by a multiple of _____

What's the rule?

Find the missing numbers.

Use a place-value chart or number line to help you.

- 3 1 more than 293 is . 4 10 more than 528 is .
- 5 100 more than 190 is . 6 20 more than 425 is .
- 7 100 more than 762 is . 8 200 more than 204 is .

Find the missing numbers.

Use a place-value chart or number line to help you.

- 9 1 less than 717 is . 10 5 less than 685 is .
- 11 10 less than 480 is . 12 30 less than 257 is .
- 13 100 less than 921 is . 14 200 less than 635 is .

Complete the number patterns.

Use place-value charts or number lines if you need to.

- 15 203, , 205, 206, 207, , 209, ,
- 16 648, 658, , 678, , , ,
- 17 721, 621, , , 321, , ,
- 18 342, , , , 338, 337, 336

Challenging Problems



Put On Your Thinking Cap!

Problem Solving

Answer the question.

Sally and Hans started counting at the same time.

Sally counted on by tens from 300.

Hans counted back by hundreds.

After six counts, they had reached the same number.

What number did Hans start counting from?



Starting now,
the assumption is:

Students know the **Commutative, Associative and Identity** Property in addition and the vocabulary associated with it

Students can compose and decompose numbers through place value and number bonds

Students can apply place value in addition with and without regrouping within 100



Do they own the place value? How will you know?

Learn

You can write numbers in word form, standard form, and expanded form.

200, 50, and 8 make 258.

258 is the **standard form** of 258.

Two hundred fifty-eight is the **word form** of 258.

$258 = 2 \text{ hundreds } 5 \text{ tens } 8 \text{ ones}$
 $= 200 + 50 + 8$

$200 + 50 + 8$ is the **expanded form** of 258.

Read 258 as
two hundred
fifty-eight.



$$278 + 386$$

In Partners:

One person use Base 10 Blocks to solve

One person draws a picture to solve

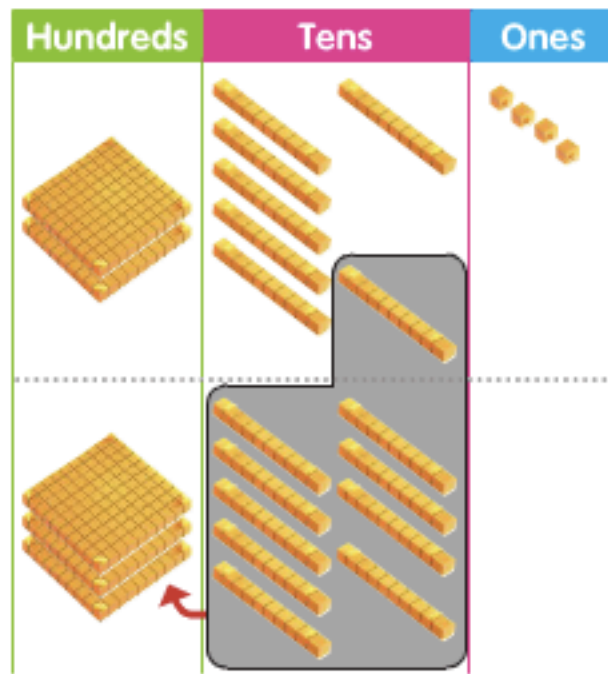
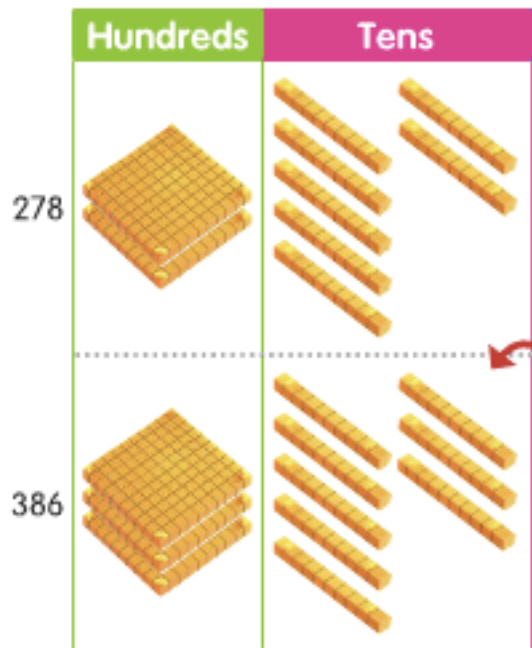
Together describe the action that is needed to complete the problem



Learn

You can add using base-ten blocks and a place-value chart to regroup ones and tens.

$$278 + 386 = ?$$



Step 2

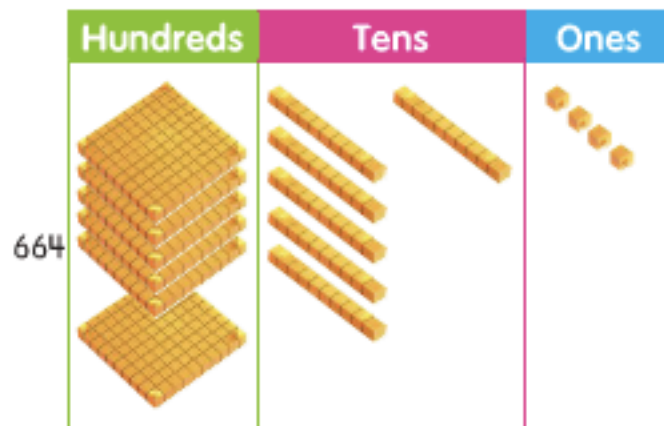
Add the tens.

$$\begin{array}{r} 2 \quad 7 \quad 8 \\ + 3 \quad 8 \quad 6 \\ \hline 6 \quad 4 \end{array}$$

1 ten + 7 tens + 8 tens
= 16 tens

Regroup the tens.

16 tens = 1 hundred 6 tens



Step 3

Add the hundreds.

$$\begin{array}{r} 2 \quad 7 \quad 8 \\ + 3 \quad 8 \quad 6 \\ \hline 6 \quad 6 \quad 4 \end{array}$$

1 hundred + 2 hundreds

+ 3 hundreds = 6 hundreds

So, $278 + 386 = 664$.

Challenging Problems

Problem of the Lesson

Complete this number sentence.

$$369 + 247 = ?$$

Show your answer using place value.

Solution:

369 = 3 hundreds 6 tens 9 ones

247 = 2 hundreds 4 tens 7 ones

369 + 247:

$$\begin{array}{r} \quad 3 \text{ hundreds} \quad 6 \text{ tens} \quad 9 \text{ ones} \\ + \quad 2 \text{ hundreds} \quad 4 \text{ tens} \quad 7 \text{ ones} \\ \hline \quad 5 \text{ hundreds} \quad 10 \text{ tens} \quad 16 \text{ ones} \\ = 5 \text{ hundreds} \quad 11 \text{ tens} \quad 6 \text{ ones} \\ = 6 \text{ hundreds} \quad 1 \text{ ten} \quad 6 \text{ ones} \\ = 616 \end{array}$$

Answer: 616

What is significant about this expanded form of regrouping?

If shown:

5 hundreds 10 tens and 16 ones,

What other addends would work?



One would never write what was not said...

$$153 + 449 = ?$$

Add the ones.

$$3 \text{ ones} + 9 \text{ ones} = \text{ } \text{ ones}$$

Regroup the ones.

$$\text{ } \text{ ones} = 1 \text{ ten } \text{ } \text{ ones}$$

Add the tens.

$$1 \text{ ten} + 5 \text{ tens} + 4 \text{ tens} = \text{ } \text{ tens}$$

Regroup the tens.

$$\text{ } \text{ tens} = 1 \text{ hundred } \text{ } \text{ tens}$$

Add the hundreds.

$$1 \text{ hundred} + 1 \text{ hundred} + 4 \text{ hundreds} = \text{ } \text{ hundreds}$$

$$\text{So, } 153 + 449 = \text{ } .$$



Challenging Problems



Put On Your Thinking Cap!

Problem Solving

Make two 3-digit numbers from the numbers below.

Use each number only once.

What are the two 3-digit numbers that give the greatest answer when you add them?



$$432 - 178$$

In Partners:

One person use Base 10 Blocks to solve

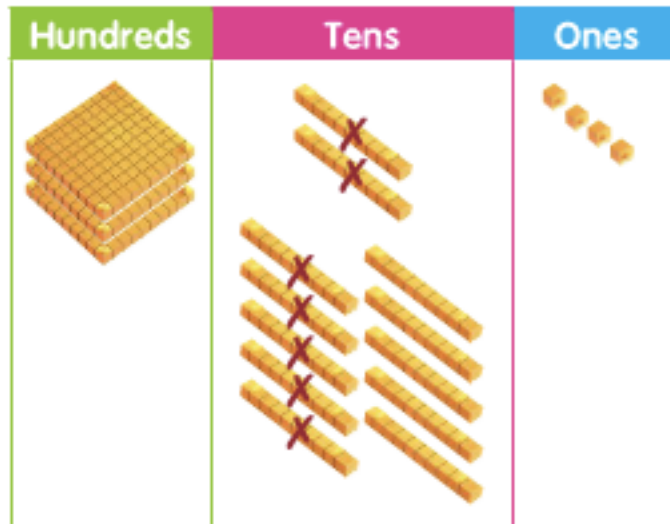
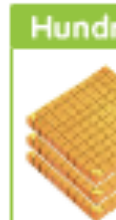
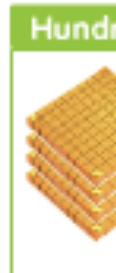
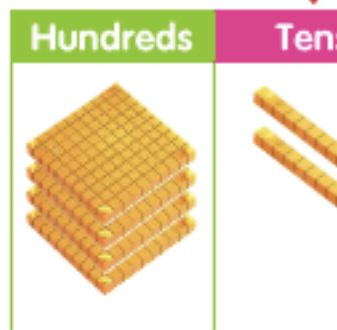
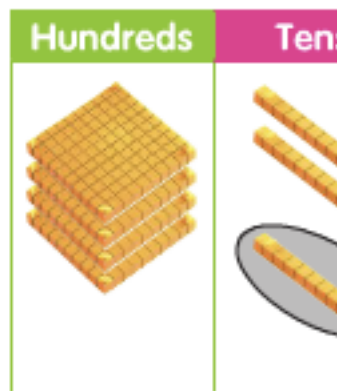
One person draws a picture to solve

Together describe the action that is needed to complete the problem



$$432 - 178 = ?$$

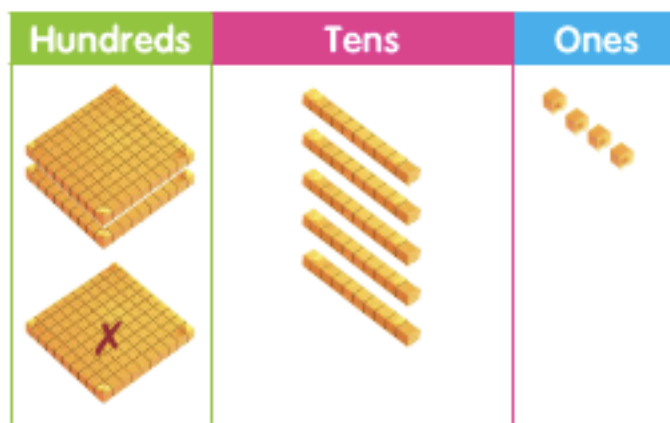
432



Subtract.

$$\begin{array}{r} ^3 ^{\frac{1}{2}} \\ 4 \ 3 \ 2 \\ - 1 \ 7 \ 8 \\ \hline 2 \ 5 \ 4 \end{array}$$

12 tens - 7 tens = 5 tens



Step 3

Subtract the hundreds.

$$\begin{array}{r} ^3 ^{\frac{1}{2}} \\ 4 \ 3 \ 2 \\ - 1 \ 7 \ 8 \\ \hline 2 \ 5 \ 4 \end{array}$$

3 hundreds - 1 hundred = 2 hundreds

So, $432 - 178 = 254$.



$$\begin{array}{r} ^3 ^{\frac{1}{2}} \\ - 1 \ 7 \ 8 \\ \hline \ 4 \end{array}$$

4 hundreds 2 tens
= 3 hundreds 12 tens

Challenging Problems

Problem of the Lesson

Suppose you have to regroup the hundreds and tens to solve these problems.

What possible digits can be placed in the boxes? Subtract to find the answer.

$$\begin{array}{r} \text{(a)} \quad 5 \ 6 \ 2 \\ - \ 3 \ \square \ \square \\ \hline \\ \hline \end{array}$$

$$\begin{array}{r} \text{(b)} \quad 9 \ 4 \ 7 \\ - \ 1 \ \square \ \square \\ \hline \\ \hline \end{array}$$



Challenging Problems

Problem of the Lesson



I am a 3-digit number. All my digits are odd numbers and each digit is different. The sum of the tens digit and hundreds digit is 6 and all the 3 digits add to 9. Who am I?



Why is this game mathematically awesome?

Hands-On Activity



WORK IN PAIRS

Players: 2
Materials:
• worksheet

STEP 1



Player 1 thinks of a 4-digit number with 1, 2, 3, and 4.
Use each digit only once.

STEP 2

Player 2 writes his or her first guess in the first row of the worksheet.

Thousands	Hundreds	Tens	Ones
1	2	4	3

STEP 3



Player 1 gives some clues.
For example, if Player 1's number is 2,314 and Player 2's guess is 1,243, Player 1 says:

- My thousands is greater than yours.
- My hundreds is greater than yours.
- My tens is less than yours.
- My ones is greater than yours.

STEP 4

Player 2 writes his or her second guess in the second row.

If his or her guess is 2,134

Player 1 will say:

- My thousands is the same as yours.
- My hundreds is greater than yours.
- My tens is less than yours.
- My ones is the same as yours.

Thousands	Hundreds	Tens	Ones
1	2	4	3
2	1	3	4

STEP 5

Thousands	Hundreds	Tens	Ones
1	2	4	3
②	1	3	④

Player 2 circles the numbers that are the same as Player 1's.
Player 2 goes on guessing until he or she gets the correct number.
Switch roles and play again!